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Macroscopic two-dimensional Wigner asymmetric islands

M Saint Jean and C Guthmann

Groupe de Physique des Solides, Universités Paris 6/ Paris 7,
Unité mixte du CNRS (UMR 75 88), 2 place Jussieu, 75251 Paris Cedex, France

E-mail: saintjea@gps.jussieu.fr

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Abstract
Using two-dimensional macroscopic Wigner islands consisting of electrostatically interacting charged balls with millimetric size, we have investigated the behaviour of a finite number of particles confined in a mesoscopic elliptic frame. In particular, we have experimentally checked the influence of the asymmetry of the confinement potential on the ground state configuration. The results obtained are compared with published numerical results.

Many significant studies have been recently dedicated to mesoscopic systems. Among these works, important interest concerns superconductor discs whose size is comparable to the coherence and the penetration lengths. In such systems, the increase of the effective penetration length due to the particular shape of these mesoscopic discs always induces type II superconductivity. The structure of the finite number of the resulting confined vortices and their transition to a giant vortex state has been intensively investigated [1, 2]. A second important field concerns the two-dimensional semiconductor quantum dots and the electron Coulomb blockade. It has been observed that, at low electron concentration, two electrons can enter the dot simultaneously. This effect has been attributed to the formation of highly symmetric electronic configurations [3, 4]. Thus, the knowledge of vortices or electron configurations in mesoscopic systems is a crucial issue to understand the properties of these systems.

Vortices or electrons confined in mesoscopic systems can be assimilated to a small number of interacting particles in a confining potential. In a previous letter [5], we presented a kind of 'macroscopic Wigner island' consisting of confined electrostatically interacting charged balls of millimetre size free to move on a plane conductor. We showed that this system is perfectly adapted to investigate the properties of such vortex or electron arrangements in circular confinement. Taking advantage of the opportunities offered by our suitable set-up, we have extended this study to explore the influence of the confinement anisotropy on the ground and metastable configurations. These experiments were also stimulated by the numerical activity concerning asymmetric systems, and they serve to complete them. In this paper, we

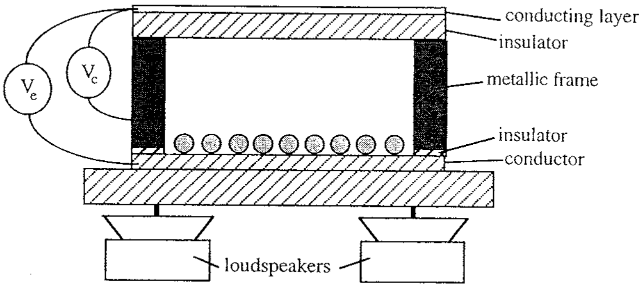

Figure 1. Experimental set-up.

present experimental results obtained for an elliptic confinement and compare them directly to recent calculations concerning vortices in mesoscopic superconductors that have an arbitrary shape varying continuously from discs to strips [2].

The experimental set-up is presented in figure 1. Our anisotropic 'Wigner islands' are constituted by stainless steel balls located on the bottom electrode of a plane horizontal capacitor, the upper electrode being a transparent conducting glass. A metallic frame with an elliptic shape is placed between the two electrodes confining the balls, the ratio b/a of the semi-axes of the ellipse being selected between 1 and 0.5. A voltage V_e, of about 1000 V, is then applied to the electrodes and maintained throughout the experiment; thus the balls become monodispersely charged. Simultaneously, a second potential V_c, tunable from 1000 to 2000 V, is applied to the confining frame. Our data presented in [5] showed that the induced confinement is parabolic.

In order to allow the system of N balls to find its energy minimum, the experimental cell is fixed to a plate that is shaken by three loudspeakers driven by white noise. This shaking produces random motion of the balls, which simulates thermal Brownian motion corresponding to an effective temperature. In order to guarantee that the ground state with minimum energy is reached, an annealing process is used. The system is initially heated up to its melting temperature (in the sense of Lindemann [6]) and cooled down again to a very low temperature. This first annealing is followed by a number of additional ones, the effective temperature maximum being at each step decreased by 10%. After these annealing cycles, the shaking is made at constant amplitude during a time varying from several minutes to several hours.

Throughout the experiment, images of the arrays of balls are recorded in real time using a CCD camera onto a VHS recorder. By analysing these very long records and measuring the time spent in each observed configuration for a set of fixed experimental parameters, we retain for 'ground state configuration' the most frequently observed state.

At low temperature, for a circular confinement the observed arrays are self-organized patterns constituted by 'concentric' shells on which the balls are located. Hereafter, the configuration in which N_i is the number of balls in the ith shell from the centre is called 'state (N_1, N_2, N_3 ...)'. Widely discussed in the literature, these peculiar structures are due to the competition between the ordering into a triangular lattice symmetry, which appears for an infinite two-dimensional coulombic system, and the symmetry imposed by the confining potential [1, 4, 7–9].

In a previous letter, we showed that with this experimental set-up the inter-ball interaction is logarithmic. To reach this conclusion, we experimentally determined the ground configurations of the balls for circular confinement, the ball number being lower than 30 [5]. For $N = 3$–5, regular polygons are formed. The first shell never exceeds five balls. For $N = 6$, a centred pentagon (1, 5) is formed. States with (1, 6), (1, 7), (1, 8) and (2, 8) patterns are obtained when N increases from seven to ten. Further increasing N to 16 causes an alternate increase of N_1 and N_2 until both shells are full, for $N_1 = 5$ and $N_2 = 11$ respectively. Beyond $N = 16$, a third shell appears; the addition of balls induces a large reorganization of the whole pattern (for instance, while the ground configuration for $N = 16$ is (5, 11), the ground configuration corresponding to $N = 17$ is (1, 5, 11)). To analyse these results, we recognized that for particular ball numbers ($N = 9$, 16, 17, 22, 24, 25, 27 and 28) the differences of configurations obtained numerically for logarithmic or coulombic potentials are energetically relevant. Comparing our results to the computed data, we concluded that our own ground state configurations are in complete agreement with the configurations obtained for a logarithmic interaction potential, suggesting that the inter-ball interaction varies more slowly than the coulombic one. Let us indicate however that, in a recent numerical study, Kong *et al* [10] have discussed this analysis. They established that the ground configurations of such systems are very sensitive to the form of the confinement potential and have suggested that our observations for $N = 9$ and 16 could correspond to a system with coulombic interparticle interaction with a confinement potential $V \sim r^n$ with $n > 2.2$ [10]. Nevertheless, as we shall see later, the similarity observed between our experimental configurations for asymmetric confinement and those calculated for explicit logarithmic inter-vortex interaction is a convincing argument for this kind of interparticle interaction. At present, we cannot explain the sound reasons for this result which could result from the finite size of the balls.

Using this experimental set-up, we have studied the case of an asymmetric confinement. This kind of confinement presents a major interest with respect to the circular case: it introduces symmetry breakdown in the system and it was interesting to look at the new configurations adapted to these conditions [11]. At first glance, the observed configurations are self-organized patterns constituted by 'concentric' shells, as for circular confinement. Indeed, whereas the local triangular symmetry of the array was maintained for circular confinement (some topological defects appearing only on the outer shell), the asymmetric confinement introduces a strong local distortion, which can completely destroy the triangular symmetry.

To illustrate this, we present in figure 2 the ground configurations obtained for $N = 4$, 5, 6 and 9, with an anisotropy ratio b/a varying from 1 to 0.5. For a given ball number N, the increase of the confinement asymmetry (b/a decrease) results at evidence in intra and/or inter-shells reorganization of the particles. For one shell arrangement, this reorganization is characterized by a particular orientation of the array with respect to the major a-axis of the ellipse. For instance, for $b/a = 0.9$, the $N = 4$ square rotates to align its diagonal along the a-axis whereas, for $N = 5$, one summit of the pentagon is pinned on this axis. Thus, for small asymmetry of the confinement, the configurations are adapted by small and smooth arrangements, by simply privileging the symmetry imposed by the confinement. In contrast, as the asymmetric ratio b/a decreases, the distortion amplifies and the triangular symmetry of the free array can be broken by the confinement. For $b/a = 0.5$, the $N = 4$ configuration results in a complete alignment of the balls whereas, for $N = 5$, the pentagon begins to rotate until it is symmetric with respect to the minor b-axis. Notice that for a five-ball system and $b/a < 0.7$ the central triangle rotates to increase the number of balls aligned with the major axis.

For two-shell structures, the asymmetric confinement can result in important rearrangement of the array. The system $N = 6$ is an example in which a two-shell ground configuration (1, 5) for circular confinement is transformed into a one-shell arrangement (6) when b/a decreases. This behaviour is also observed for $N = 7$ and 8. The situation of $N = 9$ is a little more complex. In this case, the increase of the ellipse asymmetry induces, first, a transition from the (1, 8) configuration to the (2, 7) one, this first rearrangement being followed by another which results in a return to the initial configuration (1, 8) for $b/a = 0.6$.

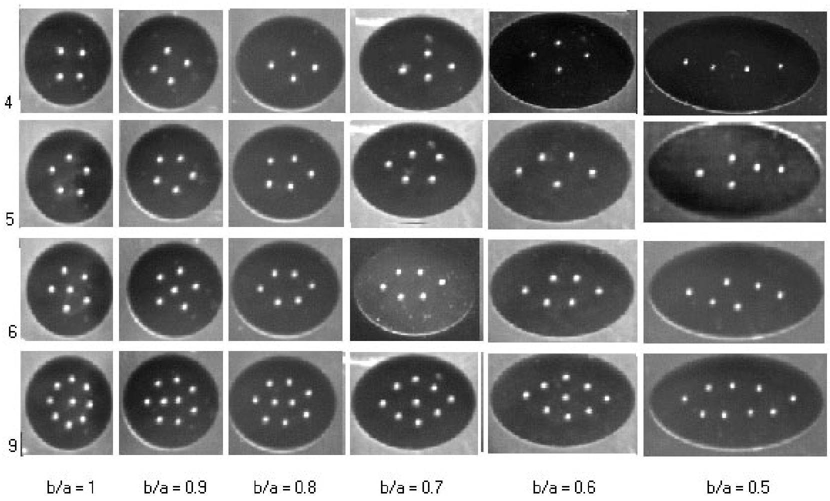

Figure 2. Ground state structures obtained for an elliptic confinement; the asymmetric ratio b/a varies from 1 to 0.5. The number of balls takes four different values: $N = 4$, 5, 6 and 9.

For systems with a higher number of balls, in which the inner shell contains more than two particles, another kind of behaviour is observed. First, the increase of the asymmetry induces no inter-shell particle exchange but simply a strong distortion of the inner shell similar to the one-shell case; an inter-shell rearrangement appears for higher distortion. For example, this is the case for $N = 12$: the ground configuration remains (3, 9) whatever the b/a ratio, the inner shell being simply elongated along a-axis while the ratio b/a decreases.

In contrast, when we examine the ground configurations obtained when N increases, some of them appear more regular than others. Figure 3 presents such configurations. These islands correspond to a 'magic number' of balls, initially introduced by Schweigert and Peeters [12], for which the confined packing is in best agreement with a hexagonal symmetry. For an axi-symmetric confinement, corresponding to a circular frame, these systems contain $N = 1 + 3p(p - 1)$ balls, p being the number of shells ($N = 7$ for $p = 2$ and 19 for $p = 3$). For these numbers, the system keeps the expected triangular lattice, the only change with respect to the ideal crystal being a slight adjustment near the outer shell. For an elliptic confinement, such regular structures are obtained for specific values of b/a and N, which have to be consistent with a triangular arrangement of particles. For instance, due to its elongated centre, the system $N = 10$ presents an elliptical (2, 8) arrangement for a circular confinement whereas this configuration corresponds to a quasi-hexagonal regular array for $b/a = 0.7$. In the same way, $N = 24$ presents a quasi-hexagonal regular pattern for $b/a = 0.8$ whereas it is not a 'magic number' for a circular confinement $b/a = 1$. For a smaller asymmetry ratio, the expected triangular configurations can be obtained; however, a slight adjustment is observed near the outer shell as illustrated for $N = 13$ with $b/a = 0.4$.

Let us give a qualitative understanding of these behaviours. As the confinement potential increases from the centre to the edge of the cell, the larger the dimension of the system, the more regular the outer shell must be in order to minimize the total energy. Thus, to minimize its energy, the system chooses the configuration which reduces the constraint imposed by the confinement on the outer shell, even if it entails some distortion in the inner shells. In other words, for a given number of balls, the reduction of the confinement potential is the most efficient process to minimize the total energy of the system.

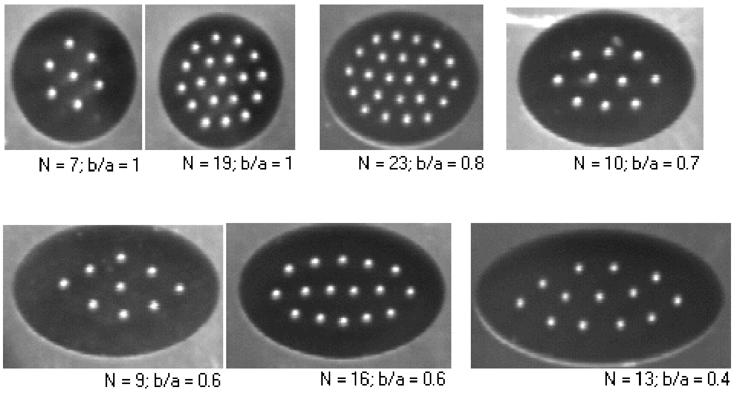

Figure 3. Regular structures and magic numbers for circular and elliptic confinements (b/a varies from 1 to 0.4).

This is particularly evident for circular confinement [5]. In the three-shell structures, we can observe that the third outer shell is roughly circular whatever the inner core. For the two-shell structures, for which the confinement is less strong since the outer shell radius is smaller, the second shell is intermediate between a circle and a polygon connecting the triangular lattice sites. For the one-shell array, the polygonal array locates the particles on a circle.

For asymmetric confinement, similar qualitative analysis can be done. The minimization process can be achieved by a rotation of a local part of the system. For instance for $N = 4$, we can observe that two balls are pinned in the direction of the minor b-axis from $b/a = 0.9$ until 0.6. In this range of the asymmetry ratio, the square is elongated along the major a-axis but the triangular array remains observable. By contrast, as $b/a = 0.5$ is reached, the largest dimension of the confinement frame imposes the rotation of the pair of balls previously oriented along the b-axis, then the four particles are aligned, the triangular feature disappears and the confinement energy decreases. A similar remark can be made for $N = 5$, b/a varying from 0.6 to 0.5. This kind of process is also observable for the two-shell structures. When the major a-axis increases, the particles take advantage of this to begin to reduce the confinement energy associated with the outer shell and the number of shells is reduced as soon as possible. For instance, the system $N = 6$, which has two shells $(1, 5)$ for $b/a = 0.9$, is transformed into one hexagonal shell for $b/a = 0.8$. We can notice that the distance between the outer shell and the confinement frame is reduced with this transition. Similar behaviour is also observed for $N = 9$ when b/a varies from 0.8 to 0.7. For larger systems, as for circular confinement, the outer shell presents a shape parallel to the frame and controls the organization of the inner shells. For instance this can be observed on the outer shells of the ground configurations obtained for $N = 16$, $b/a = 0.7$, and $N = 17$, $b/a = 0.6$, which present a regular shape roughly parallel to the confinement frame (figure 4).

As we showed in our previous paper, our experiment is also well adapted to explore the excited configurations if a convenient annealing procedure is used [5]. As the shaking is increased, raising the 'effective temperature', metastable configurations can be excited and observed. Taking advantage of this opportunity, we have explored the influence of the anisotropy of the confinement on the metastable states of the systems. As for circular confinement, the transitions between stable and metastable configurations are associated with an important reorganization of the interacting particles. Furthermore, the excitation of the ground state can be associated in anisotropic systems to local rotation. Then, we can conclude

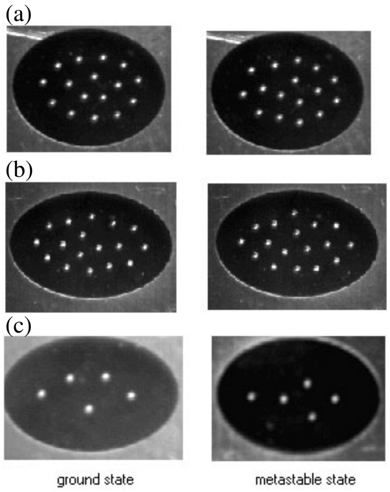

Figure 4. Ground and metastable states in elliptic confinement: (a) $N = 16$, $b/a = 0.7$; (b) $N = 17$, $b/a = 0.6$; (c) $N = 5$, $b/a = 0.6$. The inner shell structure for $N = 16$ and 17 has to be compared to the $N = 5$ configuration. For $N = 16$, the ground state corresponds to the ground $N = 5$ structure and the metastable state to the $N = 5$ metastable one while for $N = 17$ it is the opposite.

that the transition from ground to metastable states is more complex in those asymmetric systems.

To illustrate this point, let us consider the particular cases $N = 16$ ($b/a = 0.7$) and $N = 17$ ($b/a = 0.6$) in an elliptic confinement and examine the situations observed in the ground and the metastable states[1]. The configurations obtained are presented in figure 4. Even if both exhibit a two-shell configuration, (5, 11) and (5, 12) respectively, the transformation rules between the ground and metastable configurations appear really different. For instance, for $N = 16$ (figure 4(a)), the 11 balls of the outer shell in the ground state are regularly organized around an inner shell corresponding to the $N = 5$ ground state (figure 4(c)). The first metastable state corresponds to a configuration for which the inner shell corresponds to the $N = 5$ metastable state, while the outer shell does not remain regular and presents a kind of kink. In contrast, the $N = 17$ system does not follow this simple construction. For the ground state, the outer shell is almost regular whereas the inner shell corresponds to the first $N = 5$ metastable state in the same confinement (figure 4(b)). In contrast, for the metastable state, the inner shell corresponds to the $N = 5$ ground state whereas the outer shell presents an irregular shape as for the $N = 16$ excited state.

Qualitatively, we can easily understand this behaviour. When the system is mechanically excited, the energetic cost for a small displacement of one ball, due to the confinement potential, is higher for a ball located in the outer shell than for a ball in the inner shell. Thus, in order to minimize the configuration energy, the ground state corresponds always to the more regular outer shell that can be obtained, a 'zigzag' outer shell being energetically expansive (the regular to zigzag transition has been previously discussed by Schweigert *et al* [8]). The selection of the inner shell configuration is controlled by this constraint. To understand the $N = 16$ and 17 behaviours, let us remember that the $N = 5$ ground state for $b/a = 0.6$ corresponds to the natural deformation of the pentagon according to the b/a value, whereas the metastable state

[1] The $N = 16$ and 17 systems cannot be compared for the same $b/a = 0.6$ because the ground $N = 16$ configuration for the asymmetry ratio is (6/10). However, this point is not relevant for our analysis.

corresponds to the $\pi/2$ rotation of the central triangle. For $N = 16$, a regular arrangement of the 11 balls of the outer shell is consistent with this $N = 5$ ground configuration. Thus the $N = 16$ system can simultaneously satisfy the minimization of the inner and the outer shells. In contrast, an arrangement of the 12 balls of the outer shell around the $N = 5$ ground state would require defects and would induce a 'zigzag' outer shell since the central triangle repels balls out of the outer regular elliptic curve. This configuration is more expensive in energy than the configuration in which the inner shell is the metastable $N = 5$ configuration since this inner state preserves the regularity of the outer shell. Thus, to have a perfect outer shell, the system prefers to select the $N = 5$ metastable state for the inner shell. On the other hand, when the $N = 17$ system is excited, the outer shell can switch to a zigzag shell and thus offers the possibility for the inner shell to return to its ground $N = 5$ configuration.

This result can be compared to the configuration for 17 vortices interacting with a logarithmic potential, and confined within a superconducting mesoscopic ellipse, as calculated in [2]. The configuration, which is identified as the 'ground configuration', in fact corresponds exactly to our observed metastable configuration. This difference could be attributed to the difficulty to reach the minimal energy configuration, in a finite calculation time in the cases for which the energy landscape presents numerous local minima very close in energy, even if great care is used to minimize the energy. The validity of our results has recently been confirmed by the authors of [2, 13].

In conclusion, our experiments have shown that the asymmetry of the confinement induces various rearrangements of the particles, according to their number and the asymmetry ratio b/a. The rule governing these arrangements are more complex for anisotropic confinement than for a circular one. In particular, the competition between the symmetry imposed by the confinement and the triangular symmetry imposed by the inter-particle interaction results in rotating movement of a group of balls whereas this adjustment appears only on the outer shell of the configuration for circular confinement.

Moreover, the detailed study of the stable and metastable configurations obtained for anisotropic confinement shows that the stable configuration does not always correspond to the stable configuration of each shell, the case $N = 17$ and $b/a = 0.6$ being a good example of this kind of behaviour.

The obtained configurations are in complete agreement with the configurations calculated with logarithmic inter-particle interaction. These results then confirm the nature of the interaction, previously predicted for circular confinement, between balls in the present set-up.

Lastly, in a related way, we conclude that our experiment can be an excellent analogical model to determine the configurations of a finite number of interacting particles in circular or asymmetric mesoscopic systems. Indeed, our results are complementary to the many numerical studies in this topic [1, 2, 7], and our experiment is able to remove some ambiguities concerning the ground and metastable state configurations, which can appear when the configuration energies are very close. Moreover, this kind of experiment could be used in cases where the boundary conditions imposed by the shape of the confinement could not allow us simply to calculate the configurations.

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